Math 2250, Fall 2011, Quiz 13 **December 7, 2011** **R. Bruner**

In the first two problems, do not calculate. Think, instead.

1. Find the coordinates $[x^2+2x]_S$ if S is the standard basis $\{1, x, x^2\}$ of P_2.
2. Find the coordinates $[x^2+2x]_B$ if B is the basis $\{x^2+2x, x+1, 1\}$ of P_2.
3. Diagonalize the matrix

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

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1. $x^2+2x = 0\cdot 1 + 2\cdot x + 1\cdot x^2$ so $[x^2+2x]_S = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

2

2. $x^2+2x = 1(x^2+2x) + 0(x+1) + 0(1)$ so $[x^2+2x]_B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

2

3. Eigenvalues: $\begin{bmatrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{bmatrix}$ has det $(2-\lambda)^2 - 9 = \lambda^2 - 4\lambda - 5 = (\lambda+1)(\lambda-5)$

2 $\lambda = -1$: $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ iff $x = -y$ $\quad v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 5$ $\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ iff $x = y$ $\quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 2

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$$ 2

<u>OR</u>

<u>Check:</u> $\begin{bmatrix} -1 & 5 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}+\frac{5}{2} & \frac{1}{2}+\frac{5}{2} \\ \frac{1}{2}+\frac{5}{2} & -\frac{1}{2}+\frac{5}{2} \end{bmatrix} = \begin{bmatrix} 4/2 & 6/2 \\ 6/2 & 4/2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$ ok.

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1}$$

OR $= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1}$ <u>etcetera</u>.