Name: ______________________

Math 2250, Fall 2011, Quiz 12 **December 2, 2011** **R. Bruner**

1. State precisely what it means to say that λ is an eigenvalue of the linear transformation $T : V \longrightarrow V$.

2. Find the eigenvalues of

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

3. Find the eigenspace for each eigenvalue of A.

1. $Tv = \lambda v$ for some nonzero $v \in V$

2. $\det \begin{bmatrix} 2-\lambda & 1 & 2 \\ 0 & 3-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{bmatrix} = (2-\lambda)(3-\lambda)^2$ so

the eigenvalues are 2 and 3.

3. $\lambda = 2$: $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[-R_3]{-R_2-R_3} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $y=0$, $z=0$ $\quad \mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

$\lambda = 3$: $\begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-2R_2} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad -x+y=0$, $z=0$ $\quad x=y$, $y=y$, $z=0$

$\mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

<u>Checks</u>: $\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$