(Neat Version)

Name: ______________________________

Math 2250, Fall 2011, Quiz 10 **November 11, 2011** **R. Bruner**

Let $H = \{p \in P_3 \mid p(1) = 0\}$, the subspace of P_3 containing those degree 3 polynomials whose value at 1 is zero.

1. Is the following a basis of H?

$$\{x-1,\ x(x-1),\ x^2(x-1)\}$$

Hint: the fact that $p(1) = 0$ tells you something useful about the factorization of p.

2. The following set spans H. Remove vector(s) to make it into a basis.

$$\{x^2(x-1),\ x^3-1,\ x^2-1,\ x^3+x^2-2,\ x(x^2-1)\}$$

1. Yes: if $p \in H$ then $p(1) = 0$, so $p = (x-1)(ax^2+bx+c)$ for some quadratic ax^2+bx+c. Then

$$p = a(x^2(x-1)) + b(x(x-1)) + c(x-1)$$

so the set spans. It is linearly independent because $(x-1)(ax^2+bx+c) = 0 \Rightarrow ax^2+bx+c = 0 \Rightarrow a=b=c=0$.

[Multiplication of polynomials satisfies

$$AB = 0 \Rightarrow A = 0 \text{ or } B = 0.]$$

2. x^2-1 and x^3+x^2-2 are in $\text{Span}\{x^2(x-1), x^3-1\}$ (1) but $x(x^2-1)$ is not, so $\{x^2(x-1), x^3-1, x(x^2-1)\}$ is a basis for H.

[(1) $x^2-1 = x^3-1 - x^2(x-1) = x^3-1-x^3+x^2$

$x^3+x^2-2 = x^3-1 + x^2-1$ show (1), while

$x(x^2-1) \notin \text{Span}\{x^2(x-1), x^3-1\}$ since it has a nonzero linear term, but they ($x^2(x-1)$ and x^3-1) do not.]

(Less neat version)

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1. $p \in H \Rightarrow p(1) = 0$

$\Rightarrow p = (x-1)(ax^2+bx+c)$ for some quadratic ax^2+bx+c

deg 3

$\Rightarrow p = a(x^2(x-1)) + b\,x(x-1) + c(x-1) \in \text{Span}\{x^2(x-1), x(x-1), x-1\}$

Are they lin indep? $ax^2(x-1) + bx(x-1) + c(x-1)$

$= (ax^2+bx+c)(x-1)$

$= 0$

So $ax^2+bx+c = 0$

i.e. $a = b = c = 0$ so They are lin. indep as well.

Therefore they form a basis.

2. $x^2(x-1) = x^3 - x^2$ ✓

$x^3 - 1$ ✓

$x^2 - 1 = (-1)(x^3-x^2) + x^3 - 1$ throw out.

$x^3+x^2-2 = x^3-1 + x^2-1$ throw out

$x^3 - x$ need this! no x terms yet.

$$\{x^3-x^2,\ x^3-1,\ x^3-x\}$$

$$\begin{bmatrix} 0 & -1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} \Rightarrow$$ Row reduce & find pivot cols. Take corresp. polynomials.

OR Use the standard basis $\{1, x, x^2, x^3\}$ of P_3 to write the matrix of coefficients.