Name: KEY

Math 2250, Fall 2011, Quiz 1 **September 9, 2011** **R. Bruner**

Let $\mathbf{a}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 7 \\ 3 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$.

1. Write a vector equation that is equivalent to the system of linear equations

$$\begin{aligned} 2x_1 &+ 7x_2 &= 5 \\ x_1 &+ 3x_2 &= 1 \\ x_1 &+ 4x_2 &= 4 \end{aligned}$$

2. Write the corresponding augmented matrix.

3. Express $\mathbf{b}$ as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$ if it is possible.

---

(1) $x_1 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$ OR $x_1 a_1 + x_2 a_2 = b$

(2) $\left[\begin{array}{cc|c} 2 & 7 & 5 \\ 1 & 3 & 1 \\ 1 & 4 & 4 \end{array}\right]$

(3) We must row reduce (2) to do this.

$$\left[\begin{array}{cc|c} 2 & 7 & 5 \\ 1 & 3 & 1 \\ 1 & 4 & 4 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 2 & 7 & 5 \\ 1 & 4 & 4 \end{array}\right] \xrightarrow[-R_1]{-2R_1} \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow[-R_2]{-3R_2} \left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

So there is a solution, $x_1 = -8$, $x_2 = 3$.

$$-8 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 7 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}.$$