Math 2250, Fall 2011 **December 14, 2011** **R. Bruner**

Quick review topic list

1. Linear combinations as matrix products:

$$[v_1, v_2, \dots, v_n] \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ c_n \end{bmatrix} = c_1 v_1 + c_2 v_2 + \cdots + c_n v_n$$

2. Matrix form of linear equations.

3. Recognize that a matrix $A = [v_1, v_2, \dots, v_n]$ sends the coordinate vectors e_i to the vectors v_i.

4. Find the whole solution set to $Ax = b$ by row reduction.

5. The solution set of $Ax = 0$ is the null space (kernel) of A and has a basis in 1-1 correspondence with the non-pivot columns of the row reduced form of A.

6. $Ax = b$ is solvable if and only if b is in the column space (image) of A, and this has a basis consisting of the columns of A in which the pivots occur in the row reduced form of A.

7. Rank + nullity = dimension of the domain; rank is the dimension of the image (column space), nullity the dimension of the null space.

8. $\{v_1, v_2, \dots, v_n\}$ is linearly independent if and only if the null space of $A = [v_1, v_2, \dots, v_n]$ is zero. A nonzero vector in this null space gives a nontrivial linear combination of the v_i, expressing their linear dependence.

9. $\{v_1, v_2, \dots, v_n\}$ spans if and only if the column space of $A = [v_1, v_2, \dots, v_n]$ is the whole codomain (range).

10. Compute inverses by row reducing:

$$[A \mid I] \longrightarrow \left[I \mid A^{-1}\right]$$

11. Inverse of a 2 by 2 matrix.

12. $A : \mathbf{R}^n \longrightarrow \mathbf{R}^n$ is invertible if and only if it is a product of elementary matrices.

13. Subspaces and their possible dimensions. An n-dimensional subspace of an n-dimensional space is the whole space.

14. Determinants:
    (a) nonzero if and only if the matrix is invertible
    (b) expand by any row or column, or better
    (c) row reduce to triangular form, compute that determinant, then adjust for the row swaps and scalar mutiplications you did.
    (d) special cases: triangular matrices, or a row or column which is a linear combination of other rows or columns, respectively.
15. General vector spaces and linear transformations.
16. Bases.
17. Dimension of a vector space.
18. A spanning set contains a basis, which can be found by discarding vectors which are linear combinations of preceding ones in a list of the vectors in the spanning set.
19. A linearly independent set can be expanded to a basis by adding some vectors. (Easy way: add all the vectors in some basis, then use the preceding method to discard the ones you didn't really need to have added.)
20. Rank of a linear transformation = dimension of its image.
21. Coordinates with respect to a basis, $[\cdot]_{\mathcal{B}} : V \longrightarrow \mathbf{R}^n$, and linear combinations of basis elements $P_{\mathcal{B}} : \mathbf{R}^n \longrightarrow V$, and the fact that these are inverse to one another. (Here n is the number of vectors in the basis $\mathcal{B}$ of V.)
22. Eigenvalues and eigenvectors: $Av = \lambda v$.
23. Eigenvalues are roots of the characteristic polynomial $\det(A - \lambda I)$
24. Eigenvectors for eigenvalue λ are the nonzero elements of the null space of $A - \lambda I$.
25. A matrix is diagonalizable if and only if it has enough eigenvectors to span the whole space.
26. Eigenvectors for distinct eigenvalues are linearly independent.