Solutions to Test 1, M2250, F08, R. Bruner

1. (a) 5

$$2\begin{bmatrix}2\\1\\7\\3\end{bmatrix} + 3\begin{bmatrix}2\\3\\0\\1\end{bmatrix} - 5\begin{bmatrix}2\\2\\3\\2\end{bmatrix} = \begin{bmatrix}4+6-10\\2+9-10\\14+0-15\\6+3-10\end{bmatrix} = \begin{bmatrix}0\\+1\\-1\\-1\end{bmatrix}$$

(b) 5

$$\begin{bmatrix}0&0&1&0\\1&0&0&0\\0&2&0&0\\4&0&0&1\end{bmatrix}\begin{bmatrix}1&5\\2&6\\3&7\\4&8\end{bmatrix} = \begin{bmatrix}3&7\\1&5\\4&12\\8&28\end{bmatrix}$$

(c) 5

$$\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}4&3\\2&1\end{bmatrix} = \begin{bmatrix}4+4&3+2\\12+8&9+4\end{bmatrix} = \begin{bmatrix}8&5\\20&13\end{bmatrix}$$

(d)

$$\left[\begin{array}{ccc|ccc}0&1&0&1&0&0\\7&0&2&0&1&0\\3&0&1&0&0&1\end{array}\right] \xrightarrow{-2R_3} \left[\begin{array}{ccc|ccc}0&1&0&1&0&0\\1&0&0&0&1&-2\\3&0&1&0&0&1\end{array}\right] \xrightarrow{-3R_2}$$

5

$$\left[\begin{array}{ccc|ccc}1&0&0&0&1&-2\\0&1&0&1&0&0\\0&0&1&0&-3&7\end{array}\right] \quad \text{so} \quad [\;\;]^{-1} = \begin{bmatrix}0&1&-2\\1&0&0\\0&-3&7\end{bmatrix}$$

2. 5

$$\begin{bmatrix}2&2&2\\1&3&2\\7&0&3\\3&1&2\end{bmatrix}\begin{bmatrix}2\\3\\-5\end{bmatrix}$$

3. (a) 5

$$\begin{bmatrix}1&0&0&0\\0&1&7&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

(b) 5

$$\begin{bmatrix}1&&&\\&1/11&&0\\&&1&\\0&&&1\end{bmatrix}$$

(c) 5

$$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$$

5 4. (a) NOT $F[0,0]=[1,2,0]$ but $F([0,0]+[0,0]) \neq [1,2,0]+[1,20]$

(b) LINEAR

5 5. (a) NOT $[1,0]$ and $[0,1]$ are in, but $[1,1]$ is Not.

5 (b) Subspace: in fact, it is $\ker \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

<u>OR</u>: If $x_1+y_1=0$ and $y_1+z_1=0$ and if $x_2+y_2=0$ and $y_2+z_2=0$ then

$$(x_1+x_2)+(y_1+y_2)=0 \quad \text{and} \quad (y_1+y_2)+(z_1+z_2)=0$$

So W is closed under addition.

Similarly $rx_1+ry_1=r\cdot 0=0$ and $ry_1+rz_1=0$, so it is closed under scalar multiplication.

10 6.

$$\begin{aligned} x_1 + 2x_3 + 12x_5 &= 100 \\ x_2 + 3x_3 + 8x_5 &= 200 \\ x_4 + 4x_5 &= 300 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 0 \\ 300 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -12 \\ -8 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

5 7. $\ker A = \operatorname{Span}\left(\begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -12 \\ -8 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right)$

5 8.

$$\begin{bmatrix} 1 & 0 & 12 & 0 & 1 \\ 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 13 \\ 1 \\ 1 \end{bmatrix}$$

5 9.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 32 \\ 4x_1 + 5x_2 + 6x_3 &= 77 \end{aligned}$$

10. (a) $$\left[\begin{array}{ccc|c}1&1&-3&-1\\3&1&1&25\\1&0&2&13\end{array}\right] \xrightarrow[-3R_3]{-R_3} \left[\begin{array}{ccc|c}0&1&-5&-14\\0&1&-5&-14\\1&0&2&13\end{array}\right]$$

10 $$\left[\begin{array}{ccc|c}1&0&2&13\\0&1&-5&-14\\0&1&-5&-14\end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|c}1&0&2&13\\0&1&-5&-14\\0&0&0&0\end{array}\right]$$

5 (b) Yes, it is consistent: the last row is all zeros.

5 (c) No, solutions are not unique: there is a variable, x_3, with no pivot, hence a whole line of solutions.

5 (d) $$\mathrm{Im}(A) = \mathrm{Span}\left(\begin{bmatrix}1\\3\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-3\\1\\2\end{bmatrix}\right).$$