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Math 2250, Fall 2008, Test 1
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Write your answers in the bluebook, turn in the bluebook, and keep these test questions. Your answers should show your work for full credit.

1. (5 pts each) Compute

(a)

$$2 \begin{bmatrix} 2 \\ 1 \\ 7 \\ 3 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix} - 5 \begin{bmatrix} 2 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & 1 & 0 \\ 7 & 0 & 2 \\ 3 & 0 & 1 \end{bmatrix}^{-1}$$

2. (5 pts) Write the linear combination in 1(a) as a matrix times a vector.

3. (5 each) Give the 4 by 4 elementary matrices which accomplish the following elementary row operations:

(a) Add 7 times row 3 to row 2.

(b) Divide row 2 by 11.

(c) Exchange rows 2 and 3.

4. (5 each) Determine whether or not the following functions are linear. If so, just state "linear". If not, give an example which shows it is not linear.

(a) $F[x, y] = [x + 1, y + 2, x + y]$

(b) $F[x, y] = [-y, x]$

5. (5 each) Are the following subsets subspaces or not? Why?

(a) $W = \{[x, y] \mid x^2 + y^2 = 1\}$

(b) $W = \{[x, y, z] \mid x + y = 0 \text{ and } y + z = 0\}$

6. (10 pts) Find all solutions to

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 12 \\ 0 & 1 & 3 & 0 & 8 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

7. (5 pts) Find vectors which span the nullspace $\text{Ker}(A)$ of the 3×5 matrix A in the preceding problem.

8. (5 pts) Write the following equations in matrix form

$$x_1 + 12x_3 + x_5 = 13$$

$$2x_1 + x_2 + x_3 = 1$$

$$x_3 + 3x_4 + 7x_5 = 1$$

9. (5 pts) Write the following matrix equation as a system of linear equations

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 32 \\ 77 \end{bmatrix}$$

10. Consider the matrix equation

$$\begin{bmatrix} 1 & 1 & -3 \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 25 \\ 13 \end{bmatrix}$$

(a) (10 pts) Convert the augmented matrix to reduced row echelon form.

(b) (5 pts) Is the system consistent. (Why or why not?)

(c) (5 pts) Are the solutions unique? (Why or why not?)

(d) (5 pts) Find vectors which span the column space $\text{Im}(A)$.

– The End –