

R. Bruner
 Math 2250, Fall 2008, Quiz 9
 Nov 21, 2008

Use row reduction to compute

$$\det \begin{bmatrix} 2 & 1 & 4 & 3 \\ 2 & 4 & 1 & 1 \\ 4 & 3 & 4 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 4 & 3 \\ 2 & 4 & 1 & 1 \\ 4 & 3 & 4 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix} \xrightarrow[-2R_1]{-R_1} \begin{bmatrix} 2 & 1 & 4 & 3 \\ 0 & 3 & -3 & -2 \\ 0 & 1 & -4 & -3 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} \nearrow \\ \searrow \end{matrix} \begin{bmatrix} 2 & 1 & 4 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & -4 & -3 \\ 0 & 3 & -3 & -2 \end{bmatrix} \xrightarrow[-3R_2]{-R_2}$$

$$\begin{bmatrix} 2 & 1 & 4 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -5 & -4 \\ 0 & 0 & -6 & -5 \end{bmatrix} \xrightarrow{-R_4} \begin{bmatrix} 2 & 1 & 4 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -6 & -5 \end{bmatrix} \xrightarrow{+6R_3} \begin{bmatrix} 2 & 1 & 4 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\det = \underbrace{2 * 1 * 1 * 1}_{\text{from row reduced form}} * \underbrace{(-1)}_{\text{one exchange}} = -2$$

Next page: taking a short cut 

Use row reduction to compute $\det \begin{bmatrix} 2 & 1 & 4 & 3 \\ 2 & 4 & 1 & 1 \\ 4 & 3 & 4 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}$.

$$\begin{bmatrix} 2 & 1 & 4 & 3 \\ 2 & 4 & 1 & 1 \\ 4 & 3 & 4 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix} \xrightarrow[-2R_1]{-R_1} \begin{bmatrix} 2 & 1 & 4 & 3 \\ 0 & 3 & -3 & -2 \\ 0 & 1 & -4 & -3 \\ 0 & 1 & 1 & 1 \end{bmatrix} \xrightarrow[-3R_2]{-R_2} \begin{bmatrix} 2 & 1 & 4 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & -4 & -3 \\ 0 & 3 & -3 & -2 \end{bmatrix}$$

$\det = -2$

$\det = -2$

$\det = 2$

$$\begin{bmatrix} 2 & 1 & 4 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -5 & -4 \\ 0 & 0 & -6 & -5 \end{bmatrix}$$

$$\det = 2 * 1 * \det \begin{bmatrix} -5 & -4 \\ -6 & -5 \end{bmatrix}$$

$$= 2 (25 - 24)$$

$$= 2$$

and then working backward

So $\boxed{\det = -2}$