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Let F be the vector space of functions $\mathbf{R} \longrightarrow \mathbf{R}$.
For each of the following functions $T : F \longrightarrow F$

1. Determine whether or not T is linear.
2. If it is linear,
   (a) describe $\mathrm{Ker}(T)$, and
   (b) determine whether or not T is invertible.

1. $T(f) = f - 1$, regarding 1 as the constant function $1(x) = 1$.
2. $T(f) = f - f(0)$, regarding $f(0)$ as a constant function.

1. $T(f) = f-1$ is not linear

$T(2f) = 2f-1$, not $2T(f)$, which is $2f-2$.

2. $T(f) = f - f(0)$ is linear.

$T(f+g) = f+g-(f+g)(0) = f+g-f(0)-g(0) = f-f(0)+g-g(0)$

$= Tf + Tg$,

and $T(rf) = rf - (rf)(0) = r(f-f(0)) = rT(f)$.

(a) $\mathrm{Ker}(T)$ = functions f such that $f - f(0) = 0$, i.e.

$f = f(0)$, or $f(x) = f(0)$ for all x. Hence,

$\mathrm{Ker}(T)$ = constant functions.

(b) No, $\mathrm{Ker}(T) \neq 0$, so T is not invertible.