

Solutions to Quiz 6

1. We add enough vectors to be sure it spans, then row reduce to see what we can throw out

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{-2R_3} \begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{* * *} \begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{* * *} \begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Non-pivots: col 4 & 6: discard

$$\text{Basis} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2. Row reduce to see what can be discarded:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 & -1 \\ -1 & 2 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 4 & 4 & 4 & 2 \end{bmatrix} \xrightarrow{+R_1} \begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 4 & 4 & 4 & 2 \end{bmatrix}$$

Discard col's 3 & 4

$$\text{Basis} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

3. $\mathbb{R}^5 \xrightarrow{A} \mathbb{R}^3$

(a) rank $A = 2$

(i) $\dim \text{Im}(A) = 2 < 3$ so No, only those b which lie in a 2-dim subset.

(ii) nullity = $5 - \text{rank} = 5 - 2 = \underline{3}$

(b) rank $A = 3$

(i) $\dim \text{Im}(A) = 3$ so $\text{Im}(A)$ is all of \mathbb{R}^3 , so Yes

(ii) nullity = $5 - \text{rank} = 5 - 3 = \underline{2}$