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Find a basis for the span of

$$\left\{\begin{bmatrix}1\\2\\1\\1\end{bmatrix},\begin{bmatrix}2\\4\\2\\2\end{bmatrix},\begin{bmatrix}4\\7\\3\\4\end{bmatrix},\begin{bmatrix}1\\1\\0\\1\end{bmatrix}\right\}$$

and express the remaining vectors in this set as linear combinations of the basis vectors you have found.

$$\begin{bmatrix}1&2&4&1\\2&4&7&1\\1&2&3&0\\1&2&4&1\end{bmatrix}\begin{matrix}-2R_1\\-R_1\\-R_1\end{matrix}\quad\begin{bmatrix}1&2&4&1\\0&0&-1&-1\\0&0&-1&-1\\0&0&0&0\end{bmatrix}\begin{matrix}+4R_2\\*-1\\-R_2\end{matrix}\quad\begin{bmatrix}1&2&0&-3\\0&0&1&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

(* marks the pivot columns)

$$\text{Basis} = \left\{\begin{bmatrix}1\\2\\1\\1\end{bmatrix},\begin{bmatrix}4\\7\\3\\4\end{bmatrix}\right\} \quad \text{(pivot columns)}$$

$$\begin{bmatrix}2\\4\\2\\2\end{bmatrix} = 2\begin{bmatrix}1\\2\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\1\\0\\1\end{bmatrix} = -3\begin{bmatrix}1\\2\\1\\1\end{bmatrix} + \begin{bmatrix}4\\7\\3\\4\end{bmatrix}$$

Kernel:

$r_1 + 2r_2 - 3r_4 = 0$
$r_3 + r_4 = 0$

so
$r_1 = -2r_2 + 3r_4$
$r_2 = r_2$
$r_3 = -r_4$
$r_4 = r_4$
to solve

$$r_1\begin{bmatrix}1\\2\\1\\1\end{bmatrix} + r_2\begin{bmatrix}2\\4\\2\\2\end{bmatrix} + r_3\begin{bmatrix}4\\7\\3\\4\end{bmatrix} + r_4\begin{bmatrix}1\\1\\0\\1\end{bmatrix} = 0$$