

R. Bruner
Math 2250, Fall 2008, Quiz 3
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1. Write the general solution to the equations whose row-echelon augmented matrix is:

$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right] \quad \left. \begin{array}{l} x_1 + 4x_2 + 2x_5 = 1 \\ x_3 + 5x_5 = 2 \\ x_4 = 3 \end{array} \right\} \text{ so } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -2 \\ 0 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

2. Convert to reduced row-echelon form:

$$\left[\begin{array}{cccc|c} 2 & 2 & -2 & 2 & 2 \\ 1 & 1 & 1 & 5 & 3 \\ 3 & 3 & 1 & 11 & 9 \end{array} \right] \xrightarrow{* -1/2} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 5 & 3 \\ 3 & 3 & 1 & 11 & 9 \end{array} \right] \xrightarrow{\substack{-R_1 \\ -3R_1}} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 4 & 8 & 6 \end{array} \right] \xrightarrow{\substack{* 1/2 \\ -2R_2}}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{array} \right] \xrightarrow{+R_2} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{array} \right].$$

OR

$$\left[\begin{array}{cccc|c} 2 & 2 & -2 & 2 & 2 \\ 1 & 1 & 1 & 5 & 3 \\ 3 & 3 & 1 & 11 & 9 \end{array} \right] \xrightarrow{\substack{-2R_1 \\ -3R_1}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 5 & 3 \\ 2 & 2 & -2 & 2 & 2 \\ 3 & 3 & 1 & 11 & 9 \end{array} \right] \xrightarrow{\substack{* -1/4}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 5 & 3 \\ 0 & 0 & -4 & -8 & -4 \\ 0 & 0 & -2 & -4 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 5 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & -2 & -4 & 0 \end{array} \right] \xrightarrow{\substack{-R_2 \\ +2R_2}} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{array} \right].$$

Note that this system is inconsistent.