R. Bruner
Math 2250, Fall 2008, Quiz 12
Dec 5, 2008
(Take home - due Monday Dec 8)

1. Find an invertible matrix C and a diagonal matrix D such that $A = CDC^{-1}$, where

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

2. Show that $A = \begin{bmatrix} 3 & 1 \\ -1 & 5 \end{bmatrix}$ cannot be diagonalized.

1. We need eigenvalues and eigenvectors for A.

$$\det(A-\lambda I) = \det \begin{bmatrix} 2-\lambda & -1 & -1 \\ 0 & 3-\lambda & 1 \\ 0 & 1 & 3-\lambda \end{bmatrix} = (2-\lambda)\left[(3-\lambda)^2 - 1\right]$$

$= (2-\lambda)(2-\lambda)(4-\lambda)$ so eigenvalues are $2, 2, 4$.

$$\lambda_1 = 2: \quad \ker(A-2I) = \ker \begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \ker \begin{bmatrix} 0 & \overset{*}{1} & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = x\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + z\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ so $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ are independent eigenvectors for $\lambda_1 = 2$.

$\lambda_3 = 4: \quad \ker(A-3I) = \ker \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \ker \underset{\text{(No ker!)}}{\ldots}$ Hmm... Why 3? Error!

$$\ker(A-4I) = \ker \begin{bmatrix} -2 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} = \ker \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} = \ker \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x = -z$
$y = z$ in ker here, so $V_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$$C = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & & 0 \\ & 2 & \\ 0 & & 4 \end{bmatrix}.$$