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Math 2250, Fall 2008, Quiz 10
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Happy Thanksgiving. Let A be the matrix $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$.

1. Find the characteristic polynomial $\det(A - \lambda I)$.
2. Find the eigenvalues of A.
3. Find the eigenvectors of A.
4. (Optional extra credit) Express A as BDB^{-1} where B is the matrix changing to the basis of eigenvectors, and D is a diagonal matrix.

1. $\det\begin{bmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix} = (4-\lambda)(3-\lambda) - 2 = 12 - 7\lambda + \lambda^2 - 2 = \underline{\lambda^2 - 7\lambda + 10}$

$= \underline{(\lambda-2)(\lambda-5)}$

2. Eigenvalues $=$ $\underline{\underline{2}}$ and $\underline{\underline{5}}$

3. Eigenvector(s) for $\lambda_1 = 2$:

$\begin{bmatrix} 4-2 & 1 \\ 2 & 3-2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$ so $\ker(A - 2I)$ is gen by $\begin{bmatrix} x \\ y \end{bmatrix}$ with

$2x + y = 0$, or $y = -2x$, i.e. $\underline{\underline{v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}}}$.

Eigenvector for $\lambda_2 = 5$:

$\begin{bmatrix} 4-5 & 1 \\ 2 & 3-5 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$ so $v_2 = \begin{bmatrix} x \\ y \end{bmatrix}$ solves $x = y$, $\underline{\underline{v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}}$

4. $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}^{-1}$

$= \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1/3 & -1/3 \\ 2/3 & 1/3 \end{bmatrix}$