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Math 2250, Fall 2008, Quiz 1  
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1. Compute (a)  $3 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$  (b)  $3 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

2. For each of the following functions, either show it is linear or show that it is not linear (for the latter, an example will suffice).

(a)  $f(x) = 1 - 2x$

(b)  $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = 5x + 2y$

(c)  $f(x) = 0$ , a constant function.

1. (a)  $\begin{bmatrix} 6 \\ 3 \\ 9 \end{bmatrix}$  (b)  $\begin{bmatrix} 6 & -2 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

2. (a) Not linear:  $f(1) = 1 - 2(1) = 1 - 2 = -1$  but  
 $f(1+1) = f(2) = 1 - 2(2) = 1 - 4 = -3$ , not  
 $f(1) + f(1)$ , which is  $-1 + (-1) = -2$ .

(b) Linear:  $f\left(\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right) = f\left(\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}\right) = 5(x_1 + x_2) + 2(y_1 + y_2)$

and  $f\left[\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}\right] + f\left[\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\right] = 5x_1 + 2y_1 + 5x_2 + 2y_2$  is the same,

so  $f$  preserves addition. Similarly,

$f\left(c \begin{bmatrix} x \\ y \end{bmatrix}\right) = f\left[\begin{bmatrix} cx \\ cy \end{bmatrix}\right] = 5(cx) + 2(cy) = c(5x + 2y) = cf\left[\begin{bmatrix} x \\ y \end{bmatrix}\right]$

(c) Linear:  $f(x_1 + x_2) = 0$  and  $f(x_1) + f(x_2) = 0 + 0 = 0$  also,

and  $f(cx) = 0 = c \cdot 0 = cf(x)$ .