1. Complete to form a basis for $\mathbf{R}^4$:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

2. Discard vectors to get a basis for $\mathbf{R}^3$:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

3. Suppose A is a 3 by 5 matrix, i.e., a linear transformation $\mathbf{R}^5 \longrightarrow \mathbf{R}^3$.

   (a) If the rank of A is 2 then

       i. Can we solve $Ax = b$ for all vectors b in $\mathbf{R}^3$?

       ii. What is the dimension of the space of solutions to $Ax = 0$?

   (b) If the rank of A is 3 then

       i. Can we solve $Ax = b$ for all vectors b in $\mathbf{R}^3$?

       ii. What is the dimension of the space of solutions to $Ax = 0$?