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Math 2250, Fall 2008, Homework 2

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1. In each case below, find a linear transformation $f : \mathbf{R}^2 \longrightarrow \mathbf{R}$ satisfying the conditions:
   (a) $f(1,0) = 3$ and $f(0,1) = 2$
   (b) $f(1,1) = 3$ and $f(1,2) = 2$
   (c) $f(1,2) = 3$ and $f(2,1) = 2$
2. Describe the set of all linear combinations of the vectors $(2,3)$ and $(6,9)$. Explain why $(1,0)$ cannot be expressed as a linear combination of $(2,3)$ and $(6,9)$. (Hint: graph them.)
3. Express the vectors $(1,0)$ and $(0,1)$ as linear combinations of the following pairs of vectors:
   (a) $(1,2)$ and $(2,-1)$
   (b) $(1,2)$ and $(5,-4)$
   (c) $(5,7)$ and $(8,3)$
4. Draw the geometric pictures associated with the linear combinations in problem 3.
5. Let $f : \mathbf{R}^2 \longrightarrow \mathbf{R}$ be a linear transformation.
   (a) Show that there is a nonzero vector $\vec{u}_1$ such that $f(\vec{u}_1) = 0$. (Hint: start by writing f in the form $Ax + By$.)
   (b) Show that $f(c\vec{u}_1) = 0$ for every real c, where $\vec{u}_1$ is the vector from part 5a.
   (c) Show that if $f(\vec{u}_2) = 0$ for some vector $\vec{u}_2$ not parallel to $\vec{u}_1$ then $f(\vec{v}) = 0$ for every vector $v \in \mathbf{R}^2$.
6. Let $f : \mathbf{R}^2 \longrightarrow \mathbf{R}$ be the linear transformation $f(x,y) = 3x - 2y$.
   (a) Find the set N of all vectors $\vec{u}$ such that $f(\vec{u}) = 0$.
   (b) Find the set S_1 of all vectors $\vec{v}$ such that $f(\vec{v}) = 1$.
   (c) Find the set S_2 of all vectors $\vec{w}$ such that $f(\vec{w}) = 2$.
   (d) Graph the sets N, S_1 and S_2 on the same set of axes.
   (e) What can you say about the relation between these three sets?