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**Math 2250, Fall 20088**  
**Function application - extra credit**

Let  $F$  be the vector space of functions  $\mathbf{R} \rightarrow \mathbf{R}$ . For each of the following functions  $T : F \rightarrow F$ , explain why the equations listed for it are incorrect. Each of these equations appeared in proofs that  $T$  is or is not linear. In a few instances, the equation or expression may be valid but not relevant to linearity of  $T$ .

You will get 1 point extra credit for each correct explanation that you hand in on Friday November 21.

1.  $T(f) = f - 1$ , regarding 1 as the constant function  $1(x) = 1$ .

(a)  $T(f + g) = (f + g)(f - 1)$

(b)  $f(f - 1) = Tf$

(c)  $T(f + g) = (f + g)((f - 1) + (g - 1))$

(d)  $T(f + g) = (f + g)(f(x) - 1)$

(e)  $T(f + g) = f(x) + g(x) = f(x) - 1 + g(x) - 1$

(f)  $f - 1 = (f - 1)(x)$

(g)  $T(f + g) = f(x) + g(x) - 1$

(h)  $T(f(x)) = f(x) - 1(x)$

(i)  $T(f) = (f - 1)(x)$

(j)  $T(f + g) = x \cdot (f + g)(x)$

(k)  $xf(x) = f(x) - 1$

(l)  $cf(0) = f(c0)$

(m)  $T(f + g) = f - 1 + g - 1$

(n)  $f(x) - 1 = f(-1) + g(-1) = Tf + Tg(-1)$

(o)  $T(f - 1) + (g - 1)$

(p)  $T(c(f - f(0)))$

2.  $T(f) = f - f(0)$ , regarding  $f(0)$  as a constant function.

(a)  $f(x) - f(0) = Tf$

(b)  $fc - f(c0) = T(cf)$

(c)  $f - f(0) = f(0) + g(0)$