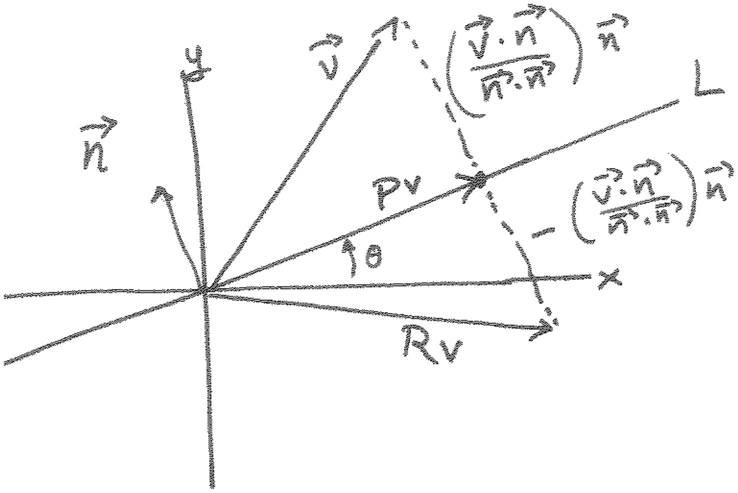
Projections, Reflections, Rotations

math 2250 F'08
R. Bruner



Projection onto the line L perpendicular to $\vec{n}$

$$P\vec{v} = \vec{v} - \left(\frac{\vec{v}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\right)\vec{n}$$

Reflection across the line L perpendicular to $\vec{n}$

$$R\vec{v} = \vec{v} - 2\left(\frac{\vec{v}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\right)\vec{n}$$

In coordinates, if $\vec{n} = \begin{bmatrix} a \\ b \end{bmatrix}$ then

$$P = \frac{1}{a^2+b^2}\begin{bmatrix} b^2 & -ab \\ -ab & a^2 \end{bmatrix} \quad \text{and} \quad R = \frac{1}{a^2+b^2}\begin{bmatrix} b^2-a^2 & -2ab \\ -2ab & a^2-b^2 \end{bmatrix}$$

If L is at angle θ to the x-axis, we may let $\vec{n} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$ and we get

$$P_\theta = \begin{bmatrix} \cos^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \quad \text{and} \quad R_\theta = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{bmatrix}$$

and rotation through θ is $T_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Algebraic properties: **Projections:** $P^2 = P$

Reflections: $R^2 = I$

Rotations: $T_\alpha T_\beta = T_{\alpha+\beta}$

$$R_\alpha R_\beta = T_{2\beta-2\alpha}$$