R. Bruner
Math 2250, Fall 2004, Homework 1
September 8, 2004

1. Which of the following functions $f : \mathbf{R} \longrightarrow \mathbf{R}$ are linear?
   (a) $f(x) = -x$
   (b) $f(x) = x/3$
   (c) $f(x) = 3/x$
   (d) $f(x) = x + 3$

2. Find (if possible) linear functions $f : \mathbf{R} \longrightarrow \mathbf{R}$ which satisfy:
   (a) $f(1) = 3$
   (b) $f(1/3) = 3$
   (c) $f(3) = 1$
   (d) $f(3) = 5$
   (e) $f(1) = 0$
   (f) $f(0) = 1$

3. Plot the graph $y = f(x)$ of the first three functions you found in the preceding problem.

4. Show that if a is a real number, then the function $f : \mathbf{R} \longrightarrow \mathbf{R}$ given by $f(x) = ax$ is linear.

5. Let $f, g : \mathbf{R} \longrightarrow \mathbf{R}$ be linear functions. Show that
   (a) $(f + g)(x) = f(x) + g(x)$ is a linear function.
   (b) $f(g(x))$ is a linear function.
   (c) $f(g(x)) = g(f(x))$.
   (d) For any real a and b, the function $h(x) = af(x) + bg(x)$ is a linear function.