1. Find a conserved quantity $E(y, y')$ for solutions to the differential equation $y'' = -4y^3$.

2. If $y(0) = 0$, $y'(0) = 2$, what are the largest and smallest values of y that ever occur?

1. $y'' = -4y^3$ has potential energy $P(y) = -\int -4y^3 \, dy = y^4$

Conserved quantity (total energy)

$$E = \tfrac{1}{2}(y')^2 + y^4$$

2. If $y(0) = 0$ and $y'(0) = 2$ then $E(0) = \tfrac{1}{2}(4) + 0 = 2$

so the solution stays within the curve

$$\tfrac{1}{2}(y')^2 + y^4 = 2$$

OR $\quad (y')^2 + 2y^4 = 4$

Max/min y occur at the "y-intercepts" $(y' = 0)$ since

$(y')^2 \geq 0$: $\qquad 0 + 2y^4 = 4$

$$y^4 = 2$$
$$y = \pm \sqrt[4]{2}$$

Min y $= -\sqrt[4]{2}$ $\qquad$ Max y $= \sqrt[4]{2}$