1. Find the values of k for which $y'' + 6y' + ky = 0$ is underdamped.

2. Find the solution to $y'' + 6y' + 8y = 0$, $y(0) = 2$, $y'(0) = 0$.

3. How many times does this solution cross $y = 0$ for $t > 0$?

1. Underdamped $\iff$ $b^2 < 4mk$ $\iff$ $6^2 < 4(1)k$ $\iff$ $k > \frac{36}{4} = 9$

$\boxed{\text{Underdamped for } k > 9}$

2. $r^2 + 6r + 8 = 0$

$(r+2)(r+4) = 0$

$y = c_1 e^{-2t} + c_2 e^{-4t}$

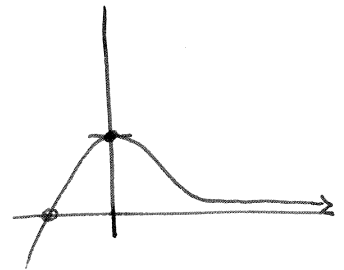$y' = -2c_1 e^{-2t} - 4c_2 e^{-4t}$

$2 = y(0) = c_1 + c_2$

$0 = y'(0) = -2c_1 - 4c_2$

So $c_1 = -2c_2$ from 2nd equation.

Then $2 = -2c_2 + c_2 = -c_2$ from 1st.

Thus $c_2 = -2$, $c_1 = 4$. $\boxed{y = 4e^{-2t} - 2e^{-4t}.}$

3. $y = 0$ $\iff$ $4e^{-2t} = 2e^{-4t}$

$\iff$ $2 = e^{-2t}$ $\Big)$ mult by $\frac{1}{2}e^{2t}$

$\iff$ $-2t = \ln 2$

$\iff$ $t = -\frac{1}{2}\ln 2 \approx -.35$

$\boxed{y \text{ Never crosses } 0 \text{ for } t > 0;}$ it crosses once

$\boxed{\text{in } t < 0.}$