

Solution

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Compute the following inverses:

$$(a) \begin{bmatrix} 1 & 0 & 4 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$(b) \begin{bmatrix} 4 & 0 & 5 \\ 2 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix}^{-1}$$

$$(a) \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow[\begin{array}{l} R_1 - 4R_3 \\ *1/5 \end{array}]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -4 \\ 0 & 1 & 0 & 0 & 1/5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(b) \left[\begin{array}{ccc|ccc} 4 & 0 & 5 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 4 & 0 & 5 & 1 & 0 & 0 \\ -2 & 2 & -2 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow[\begin{array}{l} R_1 + 2R_2 \\ R_2 - 2R_3 \end{array}]{}$$

$$\left[\begin{array}{ccc|ccc} 0 & 4 & 1 & -1 & 2 & 0 \\ -2 & 0 & -2 & -1 & 1 & -2 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow[\begin{array}{l} R_1 - 4R_3 \\ * -1/2 \end{array}]{} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -1 & 2 & -4 \\ 1 & 0 & 1 & 1/2 & -1/2 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \uparrow \\ \downarrow \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1/2 & -1/2 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 2 & -4 \end{array} \right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & -5/2 & 5 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 2 & -4 \end{array} \right]$$

$$\begin{bmatrix} 4 & 0 & 5 \\ 2 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 3/2 & -5/2 & 5 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$$