## R. Bruner Math 2150, Fall 2006, Quiz 1 September 6, 2006

1. Consider the differential equation

$$y' - 5y = 0$$

Let  $y_1(x) = e^{rx}$ . Find the value of r which makes  $y_1$  a solution of the equation.

2. Consider the differential equation

$$y' - 5y = 4x$$

Find constants m and b so that  $y_2(x) = mx + b$  solves the equation.

- 3. With these values of r, m and b, show that  $y(x) = Cy_1(x) + y_2(x)$ , i.e.,  $y(x) = Ce^{rx} + mx + b$  solves this differential equation for any constant C.
- 4. Again using these values of r, m and b, solve the differential equation with initial condition y(0) = 0. That is, find the value of C which makes both this and the differential equation true.

1. 
$$y_1 = e^{rx}$$

$$y_1' - 5y_1 = re^{rx} - 5e^{rx}$$

$$= (r - 5)e^{rx}$$
To equal 0, we need  $r = 5$ .
$$y_1 = e^{5x}$$

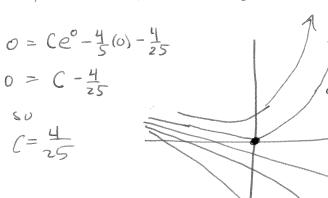
2. 
$$y_z = mx + b$$
  
 $y_z' - 5y_z = m - 5(mx + b)$   
 $= m - 5mx - 5b$   
 $= (-5m)x + (m - 5b)$   
Want = 4 x  
So  $-5m = 4$  and  $m - 5b = 0$   
Thus  $m = -\frac{4}{5}$  and  $b = \frac{1}{5}m = -\frac{4}{25}$ .  
 $y_z = -\frac{4}{5}x - \frac{4}{25}$ 

3. 
$$y = Cy. + y_2$$
  
 $= Ce^{5x} - \frac{4}{5}x - \frac{4}{25}$ . Then
$$y' - 5y = (Ce^{5x} - \frac{4}{5}x - \frac{4}{25})' - 5(Ce^{5x} - \frac{4}{5}x - \frac{4}{25})$$

$$= 5Ce^{5x} - \frac{4}{5} - 0 - 5Ce^{5x} + 4x + \frac{4}{5} = 4x, \text{ as reguired.}$$

4. Varying C gives a family of curves which are the straight line y2 - 4x - 45

plus multiples of the exponential y= 5x. Solving y(0)=0 picks out the one passing through the origin.



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