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Math 2150, Fall 2006, Quiz 1

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1. Consider the differential equation
$$y' - 5y = 0$$
Let $y_1(x) = e^{rx}$. Find the value of r which makes y_1 a solution of the equation.

2. Consider the differential equation
$$y' - 5y = 4x$$
Find constants m and b so that $y_2(x) = mx + b$ solves the equation.

3. With these values of r, m and b, show that $y(x) = Cy_1(x) + y_2(x)$, i.e., $y(x) = Ce^{rx} + mx + b$ solves this differential equation for any constant C.

4. Again using these values of r, m and b, solve the differential equation with initial condition $y(0) = 0$. That is, find the value of C which makes both this and the differential equation true.

1. $y_1 = e^{rx}$

$y_1' - 5y_1 = re^{rx} - 5e^{rx}$

$= (r-5)e^{rx}$

To equal 0, we need $r = 5$.

$y_1 = e^{5x}$

2. $y_2 = mx + b$

$y_2' - 5y_2 = m - 5(mx+b)$

$= m - 5mx - 5b$

$= (-5m)x + (m - 5b)$

Want $= 4x$

So $-5m = 4$ and $m - 5b = 0$

Thus $m = -4/5$ and $b = \frac{1}{5}m = -4/25$.

$y_2 = -\frac{4}{5}x - \frac{4}{25}$

3. $y = Cy_1 + y_2$

$= Ce^{5x} - \frac{4}{5}x - \frac{4}{25}$. Then

$y' - 5y = \left(Ce^{5x} - \frac{4}{5}x - \frac{4}{25}\right)' - 5\left(Ce^{5x} - \frac{4}{5}x - \frac{4}{25}\right)$

$= 5\cancel{Ce^{5x}} - \cancel{\frac{4}{5}} - 0 - 5\cancel{Ce^{5x}} + 4x + \cancel{\frac{4}{5}} = 4x$, as required.

4. Varying C gives a family of curves which are the straight line $y_2 = -\frac{4}{5}x - \frac{4}{25}$ plus multiples of the exponential $y_1 = 5x$. Solving $y(0) = 0$ picks out the one passing through the origin.

$0 = Ce^0 - \frac{4}{5}(0) - \frac{4}{25}$

$0 = C - \frac{4}{25}$

so

$C = \frac{4}{25}$