

12) (a)

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 6 \\ -1 & 1 & 1 & 0 & -6 \end{array} \right] \xrightarrow[\substack{-R_1 \\ +R_1}]{\substack{-R_2 \\ +R_1}} \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 6 \\ 0 & 2 & 0 & 1 & -6 \end{array} \right] \begin{array}{l} \times \frac{1}{2} \\ \times \frac{1}{2} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -3 \\ 0 & 0 & 1 & 0 & 3 \end{array} \right] \xrightarrow{-R_2 + R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & 6 \\ 0 & 1 & 0 & \frac{1}{2} & -3 \\ 0 & 0 & 1 & 0 & 3 \end{array} \right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$$

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5 (b) $\text{Dim} = 1$ (# unknowns in sol.) = $4 - 3$ (dim - rank)

5 (c) $x_1 + x_2 = (6 - \frac{1}{2}x_4) + (-3 - \frac{1}{2}x_4) = 3 - x_4 = 1 \Rightarrow x_4 = 2$

13) (a) Potential Energy = $-\int f dy = -\int -2y dy = y^2$

10 $\frac{1}{2}(y')^2 + y^2$ is conserved (constant)

5 (b) $y(0) = y'(0) = 1 \Rightarrow \frac{1}{2}(y')^2 + y^2 = \frac{3}{2}$ for all t

Max/min y occur where $y' = 0$: $\begin{array}{l} \text{Max } y = \sqrt{3/2} \\ \text{Min } y = -\sqrt{3/2} \end{array}$