6. $(s^2+9)Y = 4(1-e^{-2\pi s})\dfrac{1}{s(s^2+9)}$

$$\frac{1}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9}$$

$1 = A(s^2+9) + (Bs+C)s$

$s=0$: $\quad 1 = 9A \qquad A = 1/9$

$s=3$: $\quad 1 = 2 + 3(3B+C) \qquad -\frac{1}{3} = 3B + C$

$s=-3$: $\quad 1 = 2 - 3(-3B+C) \qquad -\frac{1}{3} = 3B - C$

so $C=0$, $B = -\frac{1}{9}$

10

$$\boxed{\begin{aligned} y &= 4\left(\tfrac{1}{9} - \tfrac{1}{9}\cos 3t\right) - 4u(t-2\pi)\left(\tfrac{1}{9} - \tfrac{1}{9}\cos(3(t-2\pi))\right) \\ &= \begin{cases} \tfrac{4}{9}(1-\cos 3t) & t < 2\pi \\ 0 & t > 2\pi \end{cases} \end{aligned}}$$

7. (a) $(4x^3+3x^2y+y^2)\,dx + (x^3+2xy+3y^2)\,dy = 0$

$\frac{\partial}{\partial y}$: $3x^2+2y$ $\qquad$ $\partial/\partial x$: $3x^2+2y$ $\qquad$ Exact

$f = \int 4x^3 + 3x^2y + y^2\,dx = x^4 + x^3y + xy^2 + g(y)$

$f_y = x^3 + 2xy + g' = x^3 + 2xy + 3y^2$ so $g'(y) = 3y^2$

10 $\boxed{F = x^4 + x^3y + xy^2 + y^3 = \text{Constant.}}$

10 (b) $\dfrac{dy}{y^2} = 12t^2$ so $-\dfrac{1}{y} = 4t^3 + C$ $\quad \boxed{y = \dfrac{-1}{4t^3+C}}$

10 (c) $u = e^{\int -7} = e^{-7t}$ $\quad \boxed{y = e^{7t}\int dt = (t+C)e^{7t}}$