1. Find the values of b for which $y'' + by' + 4y = 0$ is underdamped.

2. How many times does the solution to the initial value problem $y'' + 5y' + 4y = 0$, $y(0) = 1$, $y'(0) = -7$, cross $y = 0$ for $t > 0$?

1. Underdamped $\Longleftrightarrow$ $b^2 - 4mk < 0$

$\Longleftrightarrow$ $b^2 - 4^2 < 0$

$\Longleftrightarrow$ $-4 < b < 4$

2. First we must find the solution:

Aux eq: $r^2 + 5r + 4 = 0$ $\Rightarrow$ $r_1, r_2 = -4, -1$

General solution $y = c_1 e^{-4x} + c_2 e^{-x}$

Then $y' = -4c_1 e^{-4x} - c_2 e^{-x}$ so initial conditions

give $1 = c_1 + c_2$ Sum: $-6 = -3c_1$, or $c_1 = 2$

$-7 = -4c_1 - c_2$ Then $c_2 = 1 - c_1 = -1$.

$$y = 2e^{-4x} - e^{-x}$$

This crosses 0 when $2e^{-4x} = e^{-x}$, or $2e^{-3}$

$2 = e^{3x}$ (mult. by e^{4x})

i.e. $3x = \ln 2$, $x = \frac{1}{3} \ln 2 \approx .23$.

Ans: <u>Crosses once for $t (=x) > 0$.</u>