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Math 2150, Fall 2005, Quiz 8
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1. Find the general solution to the homogenous differential equation $y'' + 3y' + 2y = 0$.
2. Find a particular solution to the inhomogenous differential equation $y'' + 3y' + 2y = 4t^2$.
3. Find the solution to $y'' + 3y' + 2y = 4t^2$ which satisfies $y(0) = 8$ and $y'(0) = -7$.

1. Aux. equation $r^2+3r+2=0$

$(r+2)(r+1)=0$

$r=-2,-1$

$$\boxed{y = c_1 e^{-2t} + c_2 e^{-t}}$$

2. $y = At^2 + Bt + c$

$y' = 2At + B$

$y'' = 2A$

$y''+3y'+2y = 2A + 3(2At+B) + 2(At^2+Bt+c)$

$= 2At^2 + (6A+2B)t + (2A+3B+2C)$

want $= 4t^2$

So $2A = 4$ $\quad A = 2$

$6A+2B = 0$ $\quad B = -3A = -6$

$2A+3B+2C = 0$ $\quad 2C = -2A-3B = -4+18$

$= 14$

$C = 7$

$$\boxed{y_p = 2t^2 - 6t + 7}$$

3. $y = 2t^2 - 6t + 7 + c_1 e^{-2t} + c_2 e^{-t}$

$y' = 4t - 6 - 2c_1 e^{-2t} - c_2 e^{-t}$

$8 = y(0) = 7 + c_1 + c_2$

$-7 = y'(0) = -6 - 2c_1 - c_2$

$$\boxed{y = 2t^2 - 6t + 7 + e^{-t}}$$

$c_1 + c_2 = 1$

$-2c_1 - c_2 = -1$

Sum $-c_1 = 0$

So $c_1 = 0$

Then $c_2 = 1 - c_1 = 1$