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Math 2150, Fall 2005, Quiz 7
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Decide whether the following sets of vectors are linearly independent or not.

1.

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

2.

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix}.$$

3.

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}.$$

1. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \left. \vphantom{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}} \right\} \text{rank} = 3$
Dependent

2. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ clearly $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = 0$ Dependent

3. 4 vectors in a 3 dim space Dependent

2 alt. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ rank ≤ 2 , but there are 3 vectors
Dependent