R. Bruner
Math 2150, Fall 2005, Solution to Quiz 5
October 7, 2005

Find all solutions to

$$\begin{aligned} x+y+2z &= 5 \\ x+2y+2z+t &= 3 \\ 3x+4y+6z+t &= 13 \end{aligned}$$

Solution:

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 5 \\ 1 & 2 & 2 & 1 & 3 \\ 3 & 4 & 6 & 1 & 13 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \longrightarrow R_2 \\ R_3 - 3R_1 \longrightarrow R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 1 & -2 \\ 0 & 1 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} R_1 - R_2 \longrightarrow R_1 \\ \\ R_3 - R_2 \longrightarrow R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 7 \\ 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Therefore the solutions are

$$\begin{aligned} x+2z-t &= 7 \\ y+t &= -2 \end{aligned}$$

or

$$\begin{aligned} x &= 7-2z+t \\ y &= -2-t \\ z &= z \\ t &= t \end{aligned}$$

That is,

$$\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$