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Math 2150, Fall 2005, Solution to Quiz 4
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1.

$$q' + (1/RC)q = E/R$$

$$q' + 100q = 5 * 10^{-3}$$

$$q = e^{-100t} \int e^{100t} * 5 * 10^{-3}\, dt = e^{-100t}(e^{100t} * 5 * 10^{-5} + K) = 5 * 10^{-5} + Ke^{-100t}$$

Applying the initial condition $q(0) = 0$ we get

$$q(t) = 5 * 10^{-5} * (1 - e^{-100t})$$

Solving for $E_C(t) = (1/C)q(t) = 3$, i.e., $q(t) = C * 3$, we get

$$3 * 10^{-5} = 5 * 10^{-5} * (1 - e^{-100t})$$

$$2/5 = e^{-100t}$$

so $t = -\ln(2/5)/100 = 9.163 * 10^{-3}$ to 4 decimal places.

2.

$$A' = 5 * .05 - 5 * \frac{A}{200} = (1/4) - (1/40)A$$

or

$$A' + (1/40)A = 1/4$$

Thus,

$$A = e^{-t/40} \int e^{t/40}/4\, dt = e^{-t/40}(10e^{t/40} + K) = 10 + Ke^{-t/40}$$

The initial condition gives $200(.02) = 10 + K$, so $K = -6$ and $A = 10 - 6e^{-t/40}$. We want the value of t which gives a concentration of 4 %, so

$$200(.04) = 10 - 6e^{-t/40}$$

so $e^{-t/40} = (10 - 8)/6 = 1/3$ and $t = 40 \ln 3 = 43.94$ to 4 decimal places.