R. Bruner
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1.

$$\begin{aligned}\mu &= \exp(\textstyle\int 2\,dx)\\ &= e^{2x}\end{aligned}$$

$$\begin{aligned}y &= \tfrac{1}{\mu}\textstyle\int \mu q\,dx\\ &= e^{-2x}\textstyle\int e^{2x}e^{-x}\,dx\\ &= e^{-2x}\textstyle\int e^{x}\,dx\\ &= e^{-2x}(e^{x}+C)\\ &= e^{-x}+Ce^{-2x}\end{aligned}$$

$$0 = y(0) = 1 + C$$

so $C = -1$ and hence $y(x) = e^{-x} - e^{-2x}$.

2.

$$\begin{aligned}F &= \textstyle\int M\,dx + g(y)\\ &= \textstyle\int (2x+4y)\,dx + g(y)\\ &= x^2 + 4xy + g(y)\end{aligned}$$

$$\begin{aligned}F_y &= 4x + g'(y)\\ &= N\\ &= 4x + 6y\end{aligned}$$

so $g'(y) = 6y$ and $g(y) = 3y^2$. Thus the solutions are

$$x^2 + 4xy + 3y^2 = C$$

for any constant C.