R. Bruner
Math 2150, Fall 2005, Quiz 15
December 14, 2005

Use Laplace transforms to solve the differential equation

$$y'' + 4y = \begin{cases} 1 & t < \pi \\ 0 & t > \pi \end{cases}, \qquad y(0) = 0, \qquad y'(0) = 0.$$

$$(s^2+4)Y = \mathcal{L}\{1 - u(t-\pi)\} = (1 - e^{-\pi s})\frac{1}{s}$$

$$Y = (1 - e^{-\pi s})\frac{1}{s(s^2+4)}$$

$$\frac{1}{s(s^2+4)} = \frac{A}{s} + \frac{Bs}{s^2+4} + \frac{2C}{s^2+4}$$

$$1 = A(s^2+4) + Bs(s) + 2C(s)$$
$$= (A+B)s^2 + 2Cs + 4A \quad \Rightarrow \quad A = 1/4, \; B = -1/4, \; C = 0$$

$$y = \frac{1}{4} - \frac{1}{4}\cos 2t - u(t-\pi)\left(\frac{1}{4} - \frac{1}{4}\cos(2(t-\pi))\right)$$

$$= \begin{cases} \frac{1}{4} - \frac{1}{4}\cos 2t & t < \pi \\ 0 & t > \pi \end{cases}$$