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Math 2150, Fall 2005, Quiz 13
December 2, 2005

Find the Laplace transform Y of the solution to the differential equation

$$y'' - 8y' + 25y = 1, \qquad y(0) = 0, \qquad y'(0) = 1.$$

Let $Y = \mathcal{L}\{y\}$.

Then $\mathcal{L}\{y'\} = sY - y(0) = sY$

and $\mathcal{L}\{y''\} = s(sY) - y'(0) = s^2Y - 1$

Then $\mathcal{L}\{y'' - 8y' + 25y\} = \mathcal{L}\{1\}$ is

$$s^2Y - 1 - 8(sY) + 25Y = \frac{1}{s}$$

$$(s^2 - 8s + 25)Y = \frac{1}{s} + 1 = \frac{s+1}{s}$$

Answer: $$Y = \frac{s+1}{s(s^2 - 8s + 25)}$$

$s^2 - 8s + 25 = (s-4)^2 + 3^2$