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The eigenvalues of $\begin{bmatrix} 4 & -6 \\ 3 & -5 \end{bmatrix}$ are $\lambda_1 = 1$ and $\lambda_2 = -2$. (You do not need to check this.)
Find the general solution to the differential equation

$$y' = \begin{bmatrix} 4 & -6 \\ 3 & -5 \end{bmatrix} y.$$

$A - 1I = \begin{bmatrix} 3 & -6 \\ 3 & -6 \end{bmatrix}$ so $v_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ solves $3x_1 - 6x_2 = 0$

or $x_1 = 2x_2$

or $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$A - (-2)I = \begin{bmatrix} 6 & -6 \\ 3 & -3 \end{bmatrix}$ so $v_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ solves $6x_1 - 6x_2 = 0$

or $x_1 = x_2$,

or $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

The general solution is then

$$y = c_1 e^{t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

[Matrix form: $Y = \begin{bmatrix} 2e^{t} & e^{-2t} \\ e^{t} & e^{-2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$]