R. Bruner
Math 2150, Fall 2005, Quiz 10
November 9, 2005

The functions

$$y_1 = e^{3t}\begin{bmatrix}1\\1\\1\end{bmatrix}, \quad y_2 = e^{2t}\begin{bmatrix}1\\0\\0\end{bmatrix}, \quad y_3 = e^{-t}\begin{bmatrix}1\\1\\0\end{bmatrix}$$

solve the differential equation

$$y' = \begin{bmatrix}2 & -3 & 4\\0 & -1 & 4\\0 & 0 & 3\end{bmatrix} y.$$

(You do not need to check this.)

1. Show that they form a fundamental solution set.
2. Find the solution which satisfies

$$y(0) = \begin{bmatrix}1\\2\\3\end{bmatrix}.$$

1. $W = \det\begin{bmatrix}e^{3t} & e^{2t} & e^{-t}\\ e^{3t} & 0 & e^{-t}\\ e^{3t} & 0 & 0\end{bmatrix} = -e^{2t}\begin{bmatrix}e^{3t} & e^{-t}\\ e^{3t} & 0\end{bmatrix} = \boxed{e^{4t}} \neq 0$

so they are linearly independent, hence a fundamental solution set.

2. $c_1 y_1 + c_2 y_2 + c_3 y_3 = \begin{bmatrix}1\\2\\3\end{bmatrix}$ at $t=0$ gives

$\begin{bmatrix}1 & 1 & 1\\1 & 0 & 1\\1 & 0 & 0\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$. Row reduce: $\left[\begin{array}{ccc|c}1 & 1 & 1 & 1\\1 & 0 & 1 & 2\\1 & 0 & 0 & 3\end{array}\right]$

$\left[\begin{array}{ccc|c}1 & 0 & 0 & 3\\1 & 1 & 1 & 1\\1 & 0 & 1 & 2\end{array}\right] \xrightarrow[\xrightarrow[-R_1]{}]{-R_1} \left[\begin{array}{ccc|c}1 & 0 & 0 & 3\\0 & 1 & 1 & -2\\0 & 0 & 1 & -1\end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|c}1 & 0 & 0 & 3\\0 & 1 & 0 & -1\\0 & 0 & 1 & -1\end{array}\right]$

$$\boxed{y = 3e^{3t}\begin{bmatrix}1\\1\\1\end{bmatrix} - e^{2t}\begin{bmatrix}1\\0\\0\end{bmatrix} - e^{-t}\begin{bmatrix}1\\1\\0\end{bmatrix}}$$