R. Bruner
Math 2030, Winter 2016, Test 3
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1. Polar and cylindrical coordinates: Draw the diagram which shows the relation between (x, y) and (r, θ). Write the formulas for
   (a) x and y in terms of r and θ,
   (b) for $dA = dx\,dy$ in terms of r, θ, dr and $d\theta$,
   (c) for $dV = dx\,dy\,dz$ in terms of r, θ, z, dr, $d\theta$ and dz.

2. Spherical coordinates: Draw the diagram which shows the relation between (r, z) and (ρ, ϕ). Write the formulas for
   (a) r and z in terms of ρ and ϕ,
   (b) for $dA = dr\,dz$ in terms of ρ, ϕ, $d\rho$ and $d\phi$,
   (c) for $dV = dx\,dy\,dz$ in terms of ρ, θ, ϕ, $d\rho$, $d\theta$ and $d\phi$.

3. Suppose that $1 \le f(x, y) \le 4 - x - y$ for all (x, y) in the first quadrant. What can you say about $\iint_R f\,dA$, if R is the rectangle $[0, 2] \times [0, 1]$?

4. Find the area and centroid of the region in the first quadrant below $y = 4$ and above $y = x^2$.

5. Find the area and centroid of the region $0 \le r \le \sin 2\theta$ in the first quadrant. (Hint: You may use the fact that $\int_0^\pi \sin^2(u)\,du = \pi/2$.)

6. Suppose that x and y are functions of u and v in such a way that
$$\frac{\partial(x, y)}{\partial(u, v)} = u + v.$$
   Suppose that R is the region in the (x, y) plane obtained by applying this transformation to the rectangle $2 \le u \le 5$, $1 \le v \le 3$. What is the area of R?

7. Find the volume of the region described by $0 \le r \le \sin(z)$, $0 \le z \le \pi$, in cylindrical coordinates. (Recall hint from problem 5.)

8. Let R be the same region as in the preceding problem. Find the centroid of R about the z-axis. (Hint: You may use the fact that $\int_0^\pi \sin^4(u)\,du = 3\pi/8$.)

9. Find the volume of the region inside $\rho = 1 + \cos(\phi)$ in spherical coordinates.

10. Find the z-coordinate, $\bar{z}$, of the centroid of the solid in the preceding problem.

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