Math 2030, Winter 2016, Test 2

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1. (10) Find all first and second partial derivatives of $x^2y + xy^3$.

2. Let $f(x, y) = x^2 + x^2y + y^2$.

   (a) (10) Find the tangent plane to $z = f(x, y)$ at $(x, y) = (1, 2)$.

   (b) (5) In what direction does $f(x, y)$ *increase* most rapidly at $(x, y) = (1, 2)$.

   (c) (5) Find a direction in which $f(x, y)$ neither increases nor decreases at $(1, 2)$.

3. (a) (10) Find the tangent plane to the level surface $x^2yz + xy^2z + xyz^2 = 36$ at $(x, y, z) = (1, 2, 3)$.

   (b) (5) Intersecting this level surface with the plane $x+y-z = 0$ gives a curve passing through $(1, 2, 3)$. Find a vector tangent to this curve at $(1, 2, 3)$.

4. (10) Suppose that $f(x, y)$ is a function of x and y, where $x = 2s - t$ and $y = 3s + 4t$. Express Find $\partial f/\partial s$ and $\partial f/\partial t$ in terms of $\partial f/\partial x$ and $\partial f/\partial y$.

5. Suppose $C(x, y) = x^2y + xy^3$.

   (a) (5) Find the differential dC.

   (b) (5) Observe that $C(3, 2) = 42$. Estimate $C(2.99, 2.01)$ using the differential at $(x, y) = (3, 2)$.

6. (10) Let $xyz - x - y^3 - z^3 = 0$ define $z(x, y)$ as a function of x and y. Find $\partial z/\partial x$ and $\partial z/\partial y$.

7. (15) Find and classify the critical points of $x^4 - 2x^2 + y^2 - 2y$.

8. (10) Find the maximum and minimum values of $x^2 + y^3$ on the circle $x^2 + y^2 = 1$.

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