Test 1
Math 2030
W 16
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BRIEF Solutions

1. $\vec{r}(t) = (2,7,1) + t(1,2,1)$ intersects $x = y+z$ iff

$= (2+t, 7+2t, 1+t)$

$2+t = 8+3t$

i.e. $0 = 6+2t$ OR $t = -3$

$\vec{r}(-3) = (2,7,1) - (3,6,3) = \boxed{(-1,1,-2)}$

2. $(4,4,5) - (4,1,2) = (5,5,3)$

$(9,1,3) - (4,1,2) = (5,0,1)$

$$\begin{vmatrix} i & j & k \\ 5 & 5 & 3 \\ 5 & 0 & 1 \end{vmatrix} = (5, -(5-15), -25)$$

$= (5, 10, -25)$

$= 5(1,2,-5)$

$x+2y-5z = 4+2-10$

$= -4$

$\boxed{x+2y-5z+4 = 0}$

3. $\frac{(7,11,5)\cdot(1,2,-1)}{(1,2,-1)\cdot(1,2,-1)}(1,2,-1) = \frac{7+22-5}{1+4+1}(\ ) = \frac{24}{6}(1,2,-1) = \boxed{(4,8,-4) = \text{proj}}$

$\boxed{(7,11,5) = \underbrace{(4,8,-4)}_{\parallel \text{ to } (4,8,-4)} + \underbrace{(3,3,9)}_{\perp \text{ to } (1,2,-1)}}$? $3+6-9=0$ ✓

4. $z = y+1$, $y = 2x-2$

$x = x$, $y = 2x-2$, $z = 2x-1$

OR $\begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 2 & -1 & 0 \end{vmatrix} = (-1, -(0+2), -2) = (-1,-2,-2)$

OR $(1,2,2)$. A pt: $x=0 \Rightarrow y=-2, z=-1$

$\boxed{\vec{r}(t) = (0,-2,-1) + t(1,2,2)}$

5. (a) $\vec{r}(t) = (9,16,0) + t(1,3,-1)$ is in $x+3y-z-2=0$ iff

$9+t+3(16+3t)-(-t)-2=0$ i.e. $9+48+-2++t(1+9+1)=0$

$55+11t=0$, $t=-5$

$\vec{r}(-5) = (9,16,0) - (5,15,-5) = \boxed{(4,1,5)}$ (10)

(b) $5\|(1,3,-1)\| = \boxed{5\sqrt{11}}$ (5)

(c) $\vec{r}(-10) = (9,16,0) - (10,30,-10) = \boxed{(-1,-14,10)}$ (5)

6. $\vec{r}(t) = (2,0,4) + t(1,1,4) = \boxed{(2+t, t, 4+4t)}$

$\boxed{x^2 - y^2 = (2+t)^2 - t^2 = 4+4t = z}$ ✓

7: (a) $\vec{v}(t) = \langle 6, 2t+2, 3t^2+3 \rangle$

(b) $\vec{a}(t) = \langle 0, 2, 6t \rangle$

(c) $\vec{a}(1) = \langle 0,2,6 \rangle$ and $\vec{v}(1) = \langle 6,4,6 \rangle$ so $= 2\langle 3,2,3 \rangle$

$$\vec{a}_T = \frac{\langle 0,2,6\rangle \cdot \langle 6,4,6\rangle}{\langle 6,4,6\rangle \cdot \langle 6,4,6\rangle} \langle 6,4,6 \rangle = \frac{8+36}{36+16+36} \langle 6,4,6 \rangle = \frac{44}{88} \langle 6,4,6 \rangle = \langle 3,2,3 \rangle$$

$$\vec{a}_N = \vec{a} - \vec{a}_T = \langle 0,2,6 \rangle - \langle 3,2,3 \rangle = \langle -3,0,3 \rangle$$

(d) $\frac{ds}{dt}(1) = |\vec{v}(1)| = \sqrt{88} = 2\sqrt{22}$

(e) $\kappa = |\vec{a}_N| = \sqrt{9+9} = 3\sqrt{2}$

so $\kappa = 3\sqrt{2}/4.22$

(f) $\vec{r}(1) = \langle 6,3,4 \rangle$ so tangent is $\langle 6,3,4 \rangle + t\langle 6,3,6 \rangle$

(g) $y - x = t^2 + 2t - 6t = t^2 - 4t$ min at vertex $t = 2$,

$\vec{r}(2) = \langle 12, 8, 14 \rangle$

1.414
3
4.242