Name:_____

Math 2030, Winter 2016, Quiz 9
24 March 2016
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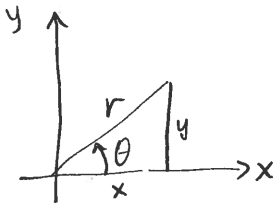
1. Polar and cylindrical coordinates: Draw the diagram which shows the relation between (x, y) and (r, θ). Write the formulas for
   (a) x and y in terms of r and θ,
   (b) for $dA = dx\,dy$ in terms of r, θ, dr and $d\theta$,
   (c) for $dV = dx\,dy\,dz$ in terms of r, θ, z, dr, $d\theta$ and dz.
2. Spherical coordinates: Draw the diagram which shows the relation between (r, z) and (ρ, ϕ). Write the formulas for
   (a) r and z in terms of ρ and ϕ,
   (b) for $dA = dr\,dz$ in terms of ρ, ϕ, $d\rho$ and $d\phi$,
   (c) for $dV = dx\,dy\,dz$ in terms of ρ, θ, ϕ, $d\rho$, $d\theta$ and $d\phi$.
3. Find the volume of the region described by $0 \le r \le z^2 - z^3$ in cylindrical coordinates.

1.

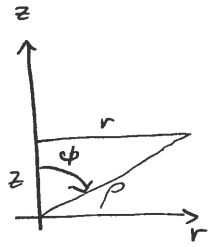


(a) $x = r\cos\theta$
$y = r\sin\theta$

(b) $dA = r\,dr\,d\theta$

(c) $dV = r\,dr\,d\theta\,dz$

2.

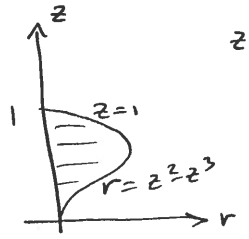


(a) $r = \rho\sin\phi$
$z = \rho\cos\phi$

(b) $dA = dr\,dz$
$= \rho\,d\rho\,d\phi$

(c) $dV = dx\,dy\,dz$
$= \rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

3.



$z^2 - z^3 = 0 \Rightarrow$
$z^2(1-z) = 0 \Rightarrow$
$z = 0$ or 1

$$\text{Vol} = \int_0^{2\pi}\int_0^1\int_0^{z^2-z^3} r\,dr\,dz\,d\theta$$

$$= 2\pi\int_0^1 \frac{1}{2}r^2\Big|_0^{z^2-z^3} dz$$

$$= \pi\int_0^1 (z^2-z^3)^2\,dz$$

$$= \pi\int_0^1 z^4 - 2z^5 + z^6\,dz$$

$$= \pi\left(\frac{1}{5} - \frac{2}{6} + \frac{1}{7}\right)$$

$$= \pi\,\frac{21-35+15}{105} = \frac{\pi}{105}$$