Name:________________________________

Math 2030, Winter 2016, Quiz 8
9 March 2016
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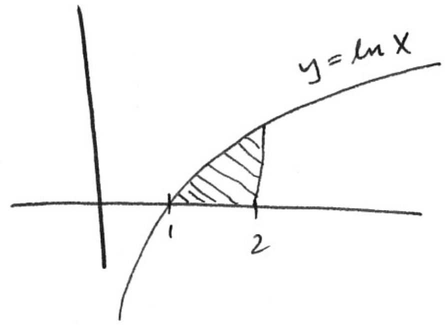
No calculators needed or allowed.

Let $R = \{(x,y) \mid 1 \le x \le 2,\ 0 \le y \le \ln x\}$. Compute $\iint\limits_R e^y \, dA$.



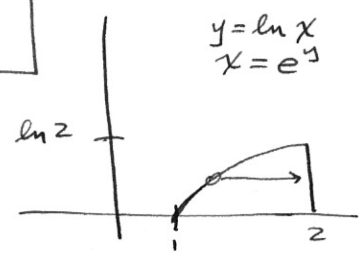
$$\int_1^2 \int_0^{\ln x} e^y \, dy \, dx = \int_1^2 e^y \Big|_{y=0}^{y=\ln x} dx$$

$$= \int_1^2 e^{\ln x} - e^0 \, dx = \int_1^2 x - 1 \, dx = \tfrac{1}{2}x^2 - x \Big|_1^2$$

$$= \tfrac{1}{2}(4) - 2 - \left(\tfrac{1}{2} - 1\right) = 2 - 2 - \left(-\tfrac{1}{2}\right) = \boxed{\tfrac{1}{2}}$$

Just FYI

If you wish to do it in the other order



$$\int_0^{\ln 2} \int_{e^y}^2 e^y \, dx \, dy = \int_0^{\ln 2} e^y (2 - e^y) \, dy$$

$$= \int_0^{\ln 2} 2e^y - e^{2y} \, dy = 2e^y - \tfrac{1}{2}e^{2y} \Big|_0^{\ln 2} = 2e^{\ln 2} - \tfrac{1}{2}e^{2\ln 2} - \left(2e^0 - \tfrac{1}{2}e^0\right)$$

$$= 2(2) - \tfrac{1}{2}(4) - \left(2 - \tfrac{1}{2}\right) = 4 - 2 - 2 + \tfrac{1}{2} = \boxed{\tfrac{1}{2}}$$

Also FYI Area $= \int_1^2 \ln x \, dx = x \ln x - x \Big|_1^2 = 2\ln 2 - 2 - (0 - 1) = \ln 4 - 1$