Name:_____

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Find and classify the critical points of $f(x,y) = x^2 + y^2 - x^2 y - y$.

we have $f_x = 2x - 2xy = 2x(1-y)$

and $f_y = 2y - x^2 - 1$.

For $f_x = 0$ we must have $x = 0$ or $y = 1$.

If $x = 0$ then $f_y = 2y - 1$, so $f_y = 0$ implies $y = \frac{1}{2}$. Then $(x,y) = (0, \frac{1}{2})$.

If $y = 1$ then $f_y = 2 - x^2 - 1 = 1 - x^2$, so $f_y = 0$ implies $x^2 = 1$, or $x = \pm 1$.

Then, $(x,y) = (1,1)$ or $(-1,1)$.

This gives three critical points: $\boxed{(0, \frac{1}{2}), \ (-1, 1), \ (1, 1)}$

To determine their type, we use the second derivative test.

	$(0, \frac{1}{2})$	$(-1, 1)$	$(1, 1)$
$f_{xx} = 2 - 2y$	1	0	0
$f_{xy} = -2x$	0	2	-2
$f_{yy} = 2$	2	2	2
$D = f_{xx}f_{yy} - f_{xy}^2$	2	-4	-4
type	min since $f_{xx} > 0$ so f is concave up	saddle	saddle