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Math 2030, Winter 2011, Quiz 5
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No calculators needed or allowed.

Let $f(x, y, z) = x^3y + y^2z$.

1. Compute the gradient ∇f.
2. Compute the directional derivative $D_{\vec{u}}f$ if $\vec{u} = (1, 1, 0)$.
3. Observe that $f(1, 3, 4) = 39$. Compute the tangent plane to $f(x, y, z) = 39$ at the point $(1, 3, 4)$.
4. Use this tangent plane to estimate the value of z for which $f(1.1, 3.1, z) = 39$.
5. Find $\partial z/\partial x$ if $z(x, y)$ is implicitly defined by the condition $f(x, y, z) = 39$.

1. $\boxed{\nabla f = (3x^2y,\ x^3+2yz,\ y^2)}$

2. $D_{\vec{u}} f = (3x^2y,\ x^3+2yz,\ y^2)\cdot(1,1,0) = \boxed{3x^2y + x^3 + 2yz}$

At $(1,3,4)$ $D_{\vec{u}} f = (9,25,9)\cdot(1,1,0) = 9+25 = \boxed{34}$ OR at (x,y,z) it is

3. $\nabla f(1,3,4) = (9,25,9)$ so the tangent plane is

$$\boxed{9(x-1) + 25(y-3) + 9(z-4) = 0}$$

4. $9(1.1-1)+25(3.1-3)+9(z-4)=0$ gives

$.9 + 2.5 + 9(z-4) = 0$, so $9(z-4) = -3.4$, $z-4 = \frac{-3.4}{9}$

and so $\boxed{z = 4 - \frac{3.4}{9}}$

5. $\frac{\partial}{\partial x}(x^3y + y^2z) = \frac{\partial}{\partial x}(39)$

$3x^2y + y^2\frac{\partial z}{\partial x} = 0$

$y^2\frac{\partial z}{\partial x} = -3x^2y \longrightarrow \boxed{\frac{\partial z}{\partial y} = \frac{-3x^2y}{y^2} = \frac{-3x^2}{y}}$