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Math 2030, Winter 2016, Quiz 3a
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No calculators needed or allowed.

Let $\vec{r}(t) = \left(\frac{2t}{t^2+2}, \frac{2}{t^2+2}, \frac{t^2}{t^2+2} \right)$.

1. Find $\lim_{t\to\pm\infty} \vec{r}(t)$.
2. Show that $\vec{r}(t)$ lies in the plane $y + z = 1$.
3. Find $\vec{r}\,'(t)$.
4. Find the maximum and minimum values of $x(t) = \vec{r}(t) \cdot \vec{i}$.

1. $\lim_{t\to\pm\infty} \left(\frac{2t}{t^2+2}, \frac{2}{t^2+2}, \frac{t^2}{t^2+2} \right) = (0, 0, 1)$

2. $y = \frac{2}{t^2+2}$ and $z = \frac{t^2}{t^2+2}$ so $y + z = \frac{2+t^2}{t^2+2} = 1$

3. $\vec{r}\,'(t) = \left(\frac{2(t^2+2) - 2t(2t)}{(t^2+2)^2}, \frac{-2(2t)}{(t^2+2)^2}, \frac{2t(t^2+2) - t^2(2t)}{(t^2+2)^2} \right)$

$= \left(\frac{4-2t^2}{(t^2+2)^2}, \frac{-4t}{(t^2+2)^2}, \frac{4t}{(t^2+2)^2} \right)$

4. Max/min $x \Rightarrow x' = 0 \Rightarrow 4 - 2t^2 = 0 \Rightarrow t^2 = 2 \Rightarrow t = \pm\sqrt{2}$

So $x(\sqrt{2}) = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$ and $x(-\sqrt{2}) = \frac{-\sqrt{2}}{2}$ are the max and min respectively, since $x \to 0$ as $t \to \pm\infty$.