Name:________________________

Math 2030, Winter 2016, Quiz 2
22 January 2016
R. Bruner

No calculators needed or allowed.

Let $\vec{r_0} = (1,6)$ and $\vec{r_1} = (4,0)$.

1. Compute the difference $\vec{v} = \vec{r_1} - \vec{r_0}$.
2. Write the parametric (i.e., explicit) equation $\vec{r}(t) = \vec{r_0} + t\vec{v}$ of the line through the points $\vec{r_0}$ and $\vec{r_1}$.
3. Find a vector $\vec{n}$ perpendicular to $\vec{v}$.
4. Write the implicit equation $\vec{n} \cdot \vec{r} + c = 0$ of the line through $\vec{r_0}$ and $\vec{r_1}$. (Hint: Simplify the equation by removing common factors if possible.)
5. Find the distance from $\vec{p_0} = (9,5)$ to this line.

1. $\vec{v} = (4,0) - (1,6) = \boxed{(3,-6)}$

2. $\vec{r}(t) = (1,6) + t(3,-6) = \boxed{(1+3t,\ 6-6t)}$

3. $\vec{n} = (6,3)$ works: $(6,3)\cdot(3,-6) = 18 - 18 = 0$.

If you forget the "trick", try this: let $\vec{n} = (a,b)$ and solve for a and b. We want $\vec{n}\cdot\vec{v} = 0$

i.e. $(a,b)\cdot(3,-6) = 0$

i.e. $3a - 6b = 0$. So $3a = 6b$ or $a = 2b$.

Thus $a=2$ if we choose $b=1$. So $\boxed{\vec{n} = (2,1) \text{ works.}}$ Note that $(6,3)$ is just 3 times this.

4. $(2,1)\cdot\vec{r} + c = 0$ should be true at $\vec{r} = \vec{r_0} = (1,6)$ so

$(2,1)\cdot(1,6) + c = 0$, or $2+6+c=0$, so $c = -8$.

Thus $\boxed{(2,1)\cdot\vec{r} - 8 = 0}$ OR $\boxed{(6,3)\cdot\vec{r} - 24 = 0}$

5. $\dfrac{(2,1)\cdot(9,5)-8}{|(2,1)|} = \dfrac{18+5-8}{\sqrt{5}} = \dfrac{15}{\sqrt{5}} = \boxed{3\sqrt{5}}$ (OR $\sqrt{45}$)

6. Write the parametric (i.e., explicit) equation $\vec{p}(t) = \vec{p_0} + t\vec{n}$ of the line through $\vec{p_0}$ in the direction of $\vec{n}$.

7. Find the intersection $\vec{p_1}$ of the two lines.
(Hint: it is probably simplest to use the equations from problems (6) and (4) rather than those from (6) and (2).)

8. Compare the distance between $\vec{p_0}$ and $\vec{p_1}$ to your answer to problem (5).

9. Sketch the results in one drawing.

6. $\vec{p}(t) = (9,5) + t(2,1) = \boxed{(9+2t,\ 5+t)}$

OR $(9,5) + t(6,3) = (9+6t,\ 5+3t)$

7. $\vec{p}(t)$ solves $(2,1)\cdot\vec{p} - 8 = 0$ if

$(2,1)\cdot(9+2t,\ 5+t) - 8 = 0$

i.e. $18 + 4t + 5 + t - 8 = 0$

or $15 + 5t = 0$. Then $t = -15/5 = -3$.

so $\vec{p}(-3) = (9-6,\ 5-3) = \boxed{(3,2) = \vec{p}_1}$

8. $\vec{p}_1 - \vec{p}_0 = (9,5) - 3(2,1) - (9,5) = -3(2,1)$

so the distance is $3|(2,1)| = 3\sqrt{5}$, agreeing with (5).

9.