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Math 2030, Winter 2016, Quiz 13x
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Let $\mathcal{S}$ be the surface $\mathbf{r}(x, y) = (x, y, x^2 + y^2)$ over the disk $x^2 + y^2 \leq 1$.
Let $\mathbf{F}(x, y, z) = (2y, 0, z)$.

1. Compute $\mathbf{n}\, dS = \dfrac{\partial \mathbf{r}}{\partial x} \times \dfrac{\partial \mathbf{r}}{\partial y}\, dA$.

2. Compute $\nabla \times \mathbf{F}$.

3. Compute $\displaystyle\iint_{\mathcal{S}} \nabla \times \mathbf{F} \cdot \mathbf{n}\, dS$.

Let $\mathbf{r}(\theta) = (\cos(\theta), \sin(\theta), 1)$, $0 \leq \theta \leq 2\pi$, be the boundary, $\partial\mathcal{S}$.

4. Compute $d\mathbf{r} = \dfrac{d\mathbf{r}}{d\theta}\, d\theta$.

5. Compute $\displaystyle\int_{\partial\mathcal{S}} \mathbf{F} \cdot d\mathbf{r}$.

1. $\vec{n}\, dS = \dfrac{\partial \vec{r}}{\partial x} \times \dfrac{\partial \vec{r}}{\partial y}\, dx\, dy = (1, 0, 2x) \times (0, 1, 2y)\, dx\, dy = (-2x, -2y, 1)\, dx\, dy$

2. $\nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2y & 0 & z \end{vmatrix} = (0, 0, -2)$

3. $\displaystyle\iint_{\mathcal{S}} \nabla \times F \cdot \vec{n}\, dS = \iint_{\text{disk}} -2\, dx\, dy = -2\, \text{Area}(\text{Disk}) = -2\pi$

4. $d\vec{r} = (-\sin\theta, \cos\theta, 0)\, d\theta$

5. $\vec{F}(\vec{r}(\theta)) = (2\sin\theta, 0, 1)$ so $\displaystyle\int_{\partial\mathcal{S}} \vec{F} \cdot d\vec{r} = \int_0^{2\pi} (2\sin\theta, 0, 1) \cdot (-\sin\theta, \cos\theta, 0)\, d\theta$

$\displaystyle = \int_0^{2\pi} -2\sin^2\theta\, d\theta = -2\pi$

(Stokes Theorem predicts that (3) and (5) will agree.)