Math 2030, Winter 2016, Quiz 12
13 April 2016
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One the the following vector fields is conservative:

$$(3x^2 + y, x + y) \qquad \text{and} \qquad (3y^2 + x, x + y)$$

1. Determine which one it is and call it $\mathbf{F}$.

2. Find a potential function for it.

3. If C is a curve starting at $(0, 2)$ and ending at $(2, 2)$, what is the value of $\int_C \mathbf{F} \cdot dr$?

1. For the first, $Q_y = 1 = P_x$

   For the second, $Q_y = 6y \neq P_x = 1$.

   So $\boxed{\text{the first is the only one that could be conservative.}}$

2. If $f_x = 3x^2 + y$ then $f = x^3 + xy + g(y)$. Then $f_y = x + g'(y)$ needs to be $x + y$, so $g'(y) = y$ and $g(y) = \frac{1}{2}y^2$ will work. So

   $\boxed{f = x^3 + xy + \frac{1}{2}y^2}$ is a potential function.

3.
$$\int_C (3x^2 + y)\,dx + (x+y)\,dy = f(2,2) - f(0,2)$$
$$= 8 + 4 + \tfrac{1}{2}(4) - \left(0 + 0 + \tfrac{1}{2}(4)\right)$$
$$= 8 + 4$$
$$\boxed{= 12}$$