Name:____________________

Math 2030, Winter 2016, Quiz 10
30 March 2016
R. Bruner

Suppose that x and y are functions of u and v in such a way that

$$\frac{\partial(x,y)}{\partial(u,v)} = u + v.$$

Suppose that R is the region in the (x, y) plane obtained by applying this transformation to the rectangle $2 \le u \le 5$, $1 \le v \le 3$. What is the area of R?

$$\int_2^5 \int_1^3 u+v \, dv \, du = \int_2^5 uv + \frac{1}{2}v^2 \Big|_{v=1}^{v=3} du = \int_2^5 3u + \frac{9}{2} - \left(u + \frac{1}{2}\right) du$$

$$= \int_2^5 2u + 4 \, du = u^2 + 4u \Big|_2^5 = 25 + 20 - (4+8)$$

$$= 45 - 12 = \boxed{33}$$

OR

$$\int_1^3 \int_2^5 u+v \, du \, dv = \int_1^3 \frac{1}{2}u^2 + uv \Big|_{u=2}^{u=5} = \int_1^3 \frac{25}{2} + 5v - (2 + 2v) \, dv$$

$$= \int_1^3 \frac{21}{2} + 3v \, dv = \frac{21v}{2} + \frac{3v^2}{2} \Big|_{v=1}^{v=3} = \frac{63}{2} + \frac{27}{2} - \left(\frac{21}{2} + \frac{3}{2}\right)$$

$$= 45 - 12 = \boxed{33}$$