Name: Key

Major/minor: _____

Math 2030, Winter 2016, Quiz 1
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R. Bruner

No calculators needed or allowed.

Let $\vec{u} = (3, 6, 9)$ and $\vec{v} = (1, 4, 3)$. Compute

1. $|\vec{v}|$,
2. $\vec{u} + 3\vec{v}$,
3. $\vec{u} - \vec{v}$,
4. the point half way between $(3, 6, 9)$ and $(1, 4, 3)$,
5. $\vec{w} = \text{proj}_{\vec{v}}\, \vec{u}$,
6. $(\vec{u} - \vec{w}) \cdot \vec{v}$,
7. the angle betweeen $(\vec{u} - \vec{w})$ and $\vec{v}$.

Answers:

1. $|\vec{v}| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{1 + 16 + 9} = \sqrt{26}$

2. $\vec{u} + 3\vec{v} = (3, 6, 9) + (3, 12, 9) = (6, 18, 18)$

3. $\vec{u} - \vec{v} = (3 - 1, 6 - 4, 9 - 3) = (2, 2, 6)$

4. $\frac{1}{2}\left[(3, 6, 9) + (1, 4, 3)\right] = \frac{1}{2}(4, 10, 12) = (2, 5, 6)$

5. $\vec{w} = \text{proj}_{\vec{v}}\, \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right)\vec{v} = \left(\frac{3 + 24 + 27}{1 + 16 + 9}\right)(1, 4, 3)$

$$= \frac{54}{26}(1, 4, 3) = \frac{27}{13}(1, 4, 3).$$

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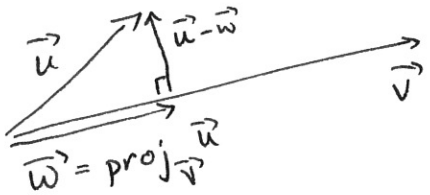
6. $\vec{u} - \vec{w}$ is perpendicular to $\vec{v}$ because $\vec{w}$ is the projection of $\vec{u}$ onto $\vec{v}$. Thus,

$$(\vec{u} - \vec{w}) \cdot \vec{v} = 0$$



OR, we can calculate: $\vec{u} - \vec{w} = \overrightarrow{(3,6,9)} - \frac{54}{26}\overrightarrow{(1,4,3)}$

so

$$\begin{aligned}(\vec{u} - \vec{w}) \cdot \vec{v} &= \vec{u} \cdot \vec{v} - \vec{w} \cdot \vec{v} \\ &= \overrightarrow{(3,6,9)} \cdot \overrightarrow{(1,4,3)} - \frac{54}{26}\overrightarrow{(1,4,3)} \cdot \overrightarrow{(1,4,3)} \\ &= (3 + 24 + 27) - \frac{54}{26}(1 + 16 + 9) \\ &= 54 - \frac{54}{26} \cdot 26 = 0 .\end{aligned}$$

7. Since they are perpendicular (the dot product is 0) the angle is $\pi/2$.