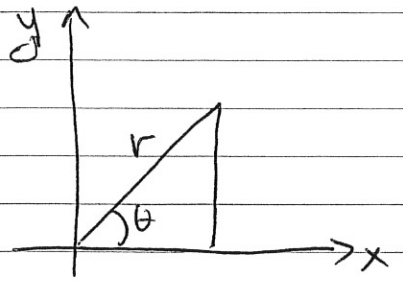
Cylindrical Coordinates:

Replace (x,y) by (r,θ) and leave z - alone.
Useful in symmetry about an axis



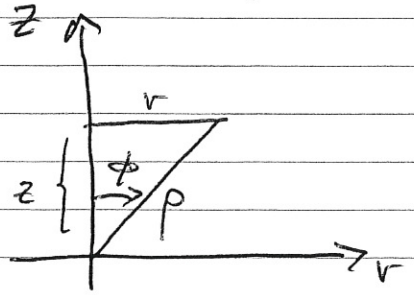
$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$dx\,dy = r\,dr\,d\theta \qquad (1)$$

$$dx\,dy\,dz = r\,dr\,d\theta\,dz$$

Spherical Coordinates

Replace (r,z) by (ρ,ϕ) and leave θ alone.
Useful in symmetry about a point.



$$z = \rho\cos\phi$$
$$r = \rho\sin\phi \qquad (2)$$

$$dr\,dz = \rho\,d\rho\,d\phi \qquad \text{by (1)}$$

So $\quad dx\,dy\,dz = r\,dr\,d\theta\,dz$

$$= r\rho\,d\rho\,d\theta\,d\phi$$

$$= \rho^2\sin\phi\,d\rho\,d\theta\,d\phi \qquad \text{by (2)}$$

r = distance to z-axis $\qquad r^2 = x^2+y^2$

ρ = distance to origin $\qquad \rho^2 = r^2+z^2 = x^2+y^2+z^2$